Announcements

1) Math Major Pizza/t-shirt event in Math Library (CB 2047) from noon to 1:30 on Monday

2) Exam 1 Thursday next week, review on Tuesday

Linear Dependence: Two functions f and g, defined on an interval I, are said to be **linearly dependent** if there is a real number c such that

$$\boxed{f(x) = c\,g(x)}$$

for all x in I.

Example 1: (distinct roots)

Solve

$$y''(t) + y'(t) - 6y(t) = 0,$$

$$y(0) = 1, \quad y'(0) = 0.$$

Suppose $y(t) = e^{rt}$ for some r.

Then again,

$$y'(t) = re^{rt},$$

$$y''(t) = r^2e^{rt},$$ and we get

$$r^2e^{rt} + re^{rt} - 6e^{rt} = 0$$

Cancel e^{rt} to get

$$r^2 + r - 6 = 0$$

$$\frac{e^{rt}(r^2+r-6)}{e^{rt}} = \frac{0}{e^{rt}}$$

$$\rightarrow r^2+r-6=0$$

$$(r+3)(r-2)=0$$

$$r=-3,\ 2$$

$$\boxed{f_1(t)=e^{-3t} \text{ and } f_2(t)=e^{2t} \text{ are solutions}}$$

We get that

$$c_1 e^{-3t} + c_2 e^{2t} = y(t)$$

is a solution for all constants c_1, c_2.

Use initial conditions to determine c_1, c_2:

$y(0) = 1$, so

$$c_1 + c_2 = 1$$

Since $y'(0)=0$,

$$0 = \left(c_1 e^{-3t} + c_2 e^{2t}\right)' \text{ at } t=0$$

$$= \left(-3c_1 e^{-3t} + 2c_2 e^{2t}\right) \text{ at } t=0$$

$$= -3c_1 + 2c_2$$

We get the equations

$$c_1 + c_2 = 1$$

$$-3c_1 + 2c_2 = 0.$$

Substituting $c_1 = 1 - c_2$ into the second equation,

$$-3 + 3c_2 + 2c_2 = 0,$$

so $c_2 = 3/5$ and

$$c_1 = 1 - c_2 = 2/5$$

Final answer:

$$\boxed{y(t) = \frac{2}{5}e^{-3t} + \frac{3}{5}e^{2t}}$$

Q: How do we know what solutions look like?

Nice Result: Given a second-order, linear differential equation that is homogeneous with constant coefficients, all solutions are either of the form